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Path following of an underactuated ship in restricted waters using CLOS and compensated LQR methods

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1. Introduction



➤ Ship maneuvering motion in restricted waters

- **Shallow water** and **bank effects** are two critical factors in such scenarios.
- Environmental factors significantly influence **ship maneuverability**.
- Notable changes in ship maneuverability present additional challenges to **motion control**.



Ships in restricted waters (from the internet)

1. Introduction



➤ Ship maneuvering motion model in restricted waters

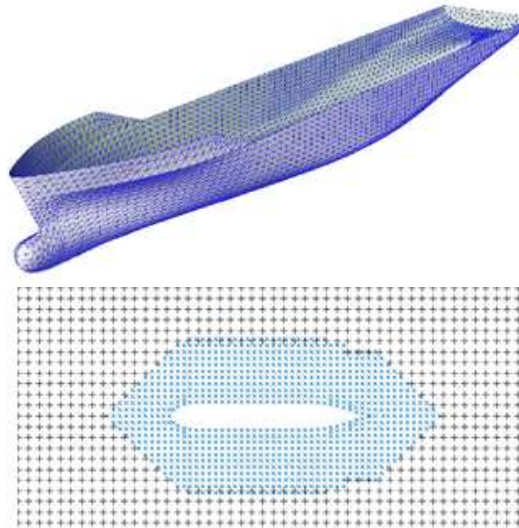
- Before the development of the control method, it is essential to first establish a **ship maneuvering motion model** considering restricted-water impacts.

Model test methods



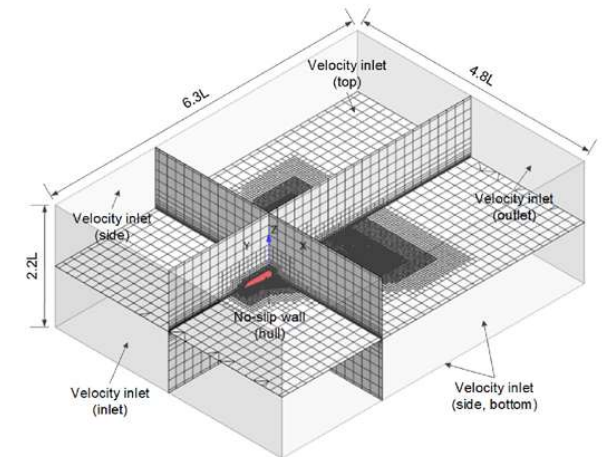
MASHCON2022
benchmark model
tests

Potential flow methods



Hull surface panels and free
surface node distributions
(Shi et al., 2024)

CFD methods



CFD simulations
(Guo et al., 2022)

1. Introduction



➤ Advanced control methods

- Shallow water and bank effects affect ship maneuverability in restricted waters, driving the need for **advanced control** solutions.

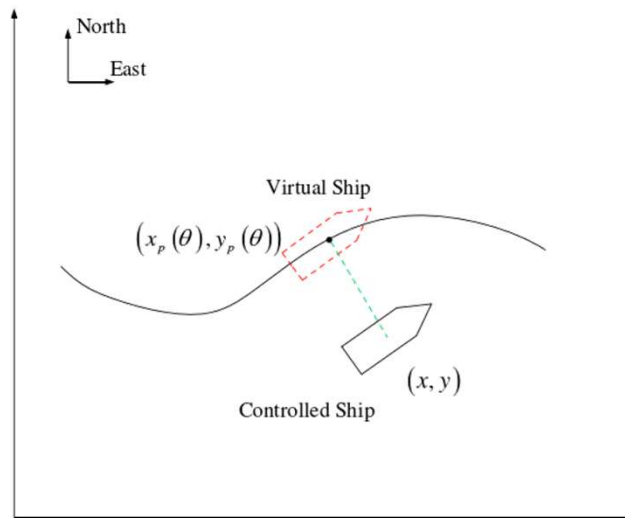
Control Method	Advantages	Disadvantages
PID control	Simple, real-time performance, robustness	Difficult parameter tuning, limited effectiveness for complex systems
Lyapunov-based control	Stability guarantee, wide applicability, strong robustness	Complex design, high computational cost, potential conservatism
Optimal control	Global optimality, strong applicability, high flexibility	Computational complexity, poor real-time performance, model dependency

1. Introduction



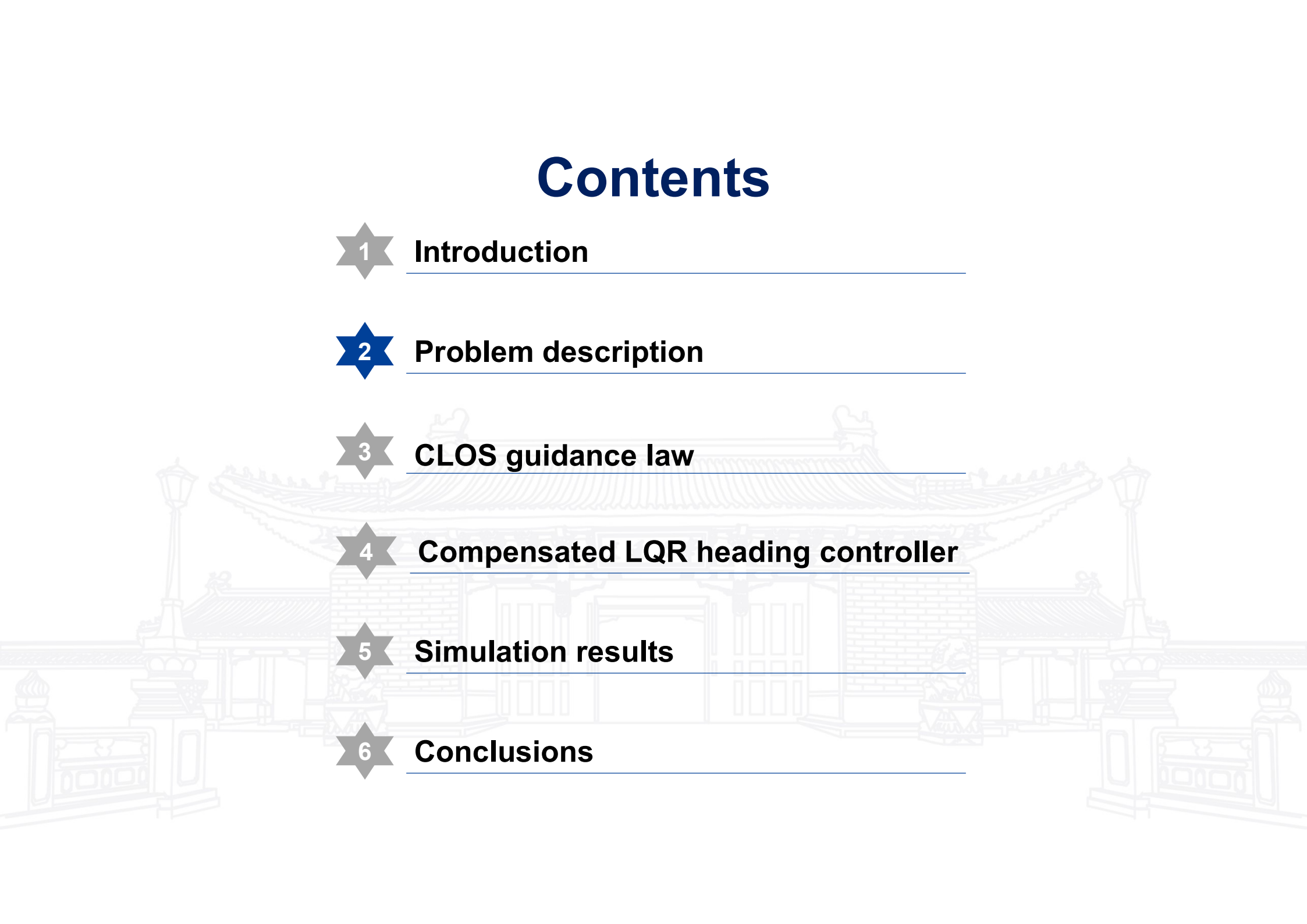
➤ Ship path following

- Among various ship motion control in restricted waters, **path following** is regarded as a fundamental task that plays an important role in **berthing/unberthing**, **docking**, and **collision avoidance** with static structures and other ships.



Ships path following

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2. Problem description

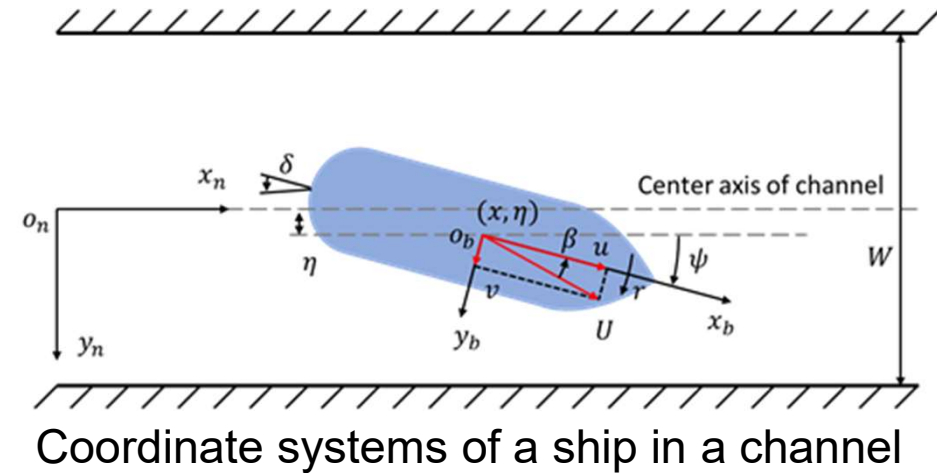
➤ Equations of ship motion

- A nondimensional **Maneuvering Model Group (MMG) model** is developed specifically for ship motions in channels (Sano et al., 2014):

$$\begin{cases} (m' + m'_{22})\dot{v}' + (x'_G m' + m'_{26})\dot{r}' + (m' + m'_{11})r' = Y' \\ (I'_z + x'^2_G m' + m'_{66})\dot{r}' + (x'_G m' + m'_{26})\dot{v}' + x'_G m' r' = N' \end{cases}$$

$$\begin{cases} Y' = Y'_0 + Y'_\beta \beta + Y'_r r' + Y'_\eta \eta' + Y'_\delta \delta + Y'_{\beta\beta\beta} \beta^3 + Y'_{\beta\eta\eta} \beta \eta'^2 \\ \quad + Y'_{\beta\beta\eta} \beta^2 \eta' + Y'_{\eta\eta\eta} \eta'^3 + Y'_{\eta\eta\delta} \eta'^2 \delta + Y'_{\beta\delta\delta} \beta \delta^2 + Y'_{\beta\beta\delta} \beta^2 \delta \\ N' = N'_0 + N'_\beta \beta + N'_r r' + N'_\eta \eta' + N'_\delta \delta + N'_{\beta\beta\beta} \beta^3 + N'_{\beta\eta\eta} \beta \eta'^2 \\ \quad + N'_{\beta\beta\eta} \beta^2 \eta' + N'_{\eta\eta\eta} \eta'^3 + N'_{\eta\eta\delta} \eta'^2 \delta + N'_{\beta\delta\delta} \beta \delta^2 + N'_{\beta\beta\delta} \beta^2 \delta \end{cases}$$

$$\begin{cases} \dot{x} = u \cos \psi - v \sin \psi \\ \dot{\eta} = u \sin \psi + v \cos \psi \\ \dot{\psi} = r \end{cases} \longrightarrow \begin{cases} \dot{x}' = 1 \\ \dot{\eta}' = \psi + v' \\ \dot{\psi}' = r' \end{cases}$$



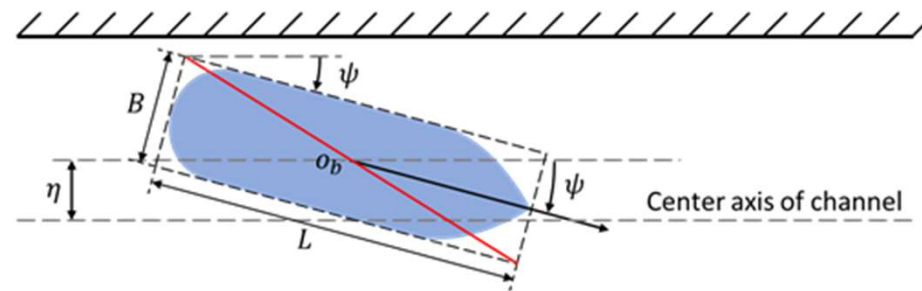
2. Problem description



➤ Collision-avoidance criteria

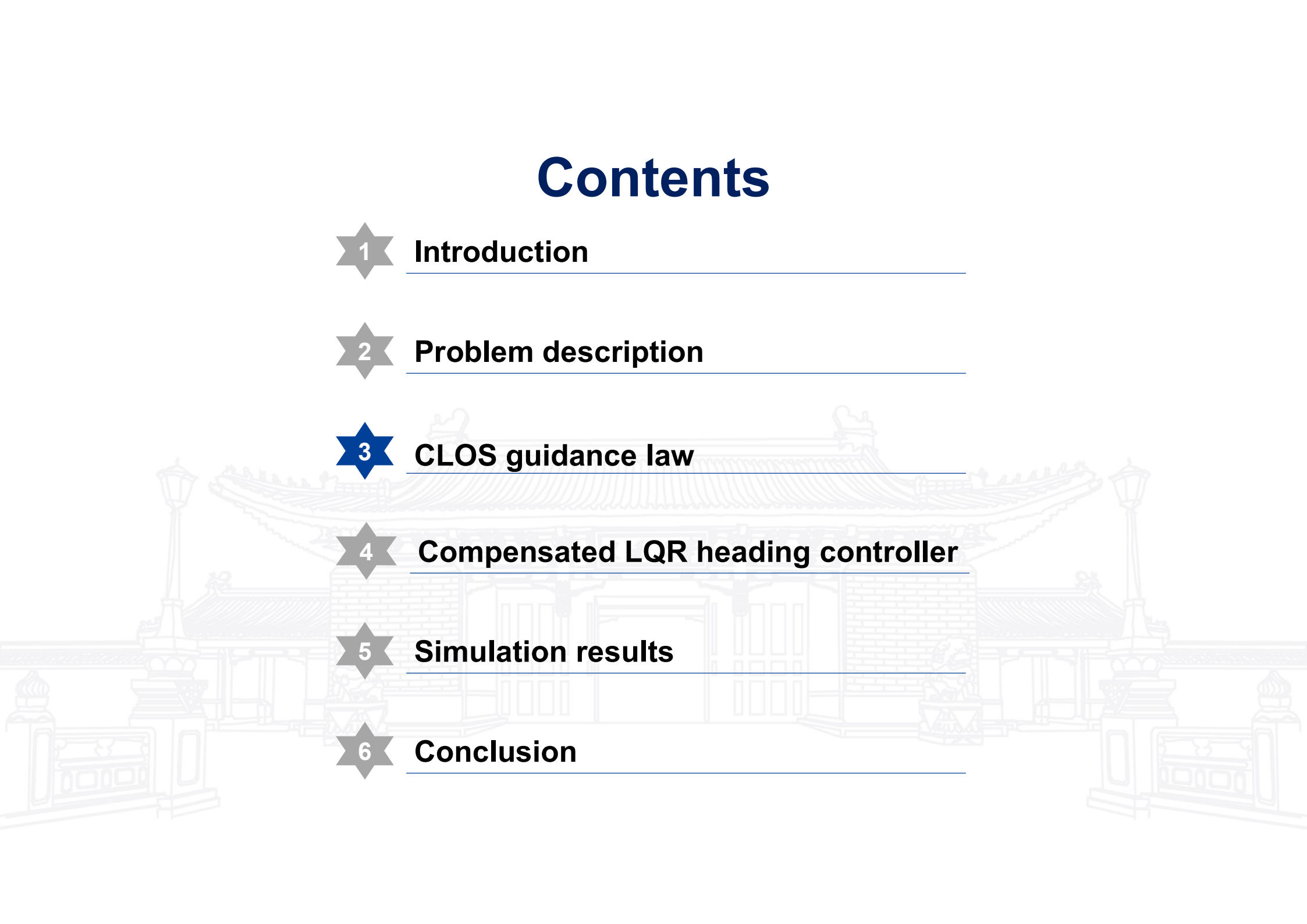
- To assess the potential for the ship collisions with the channel during the navigation, the following simplified **collision-avoidance criteria** has been established:

$$f(\eta, \psi) = |\eta| + \sin \left[|\psi| + \arctan \left(\frac{B}{L} \right) \right] \frac{\sqrt{L^2 + B^2}}{2} < \frac{W}{2}$$



Principle of the collision-avoidance conditions

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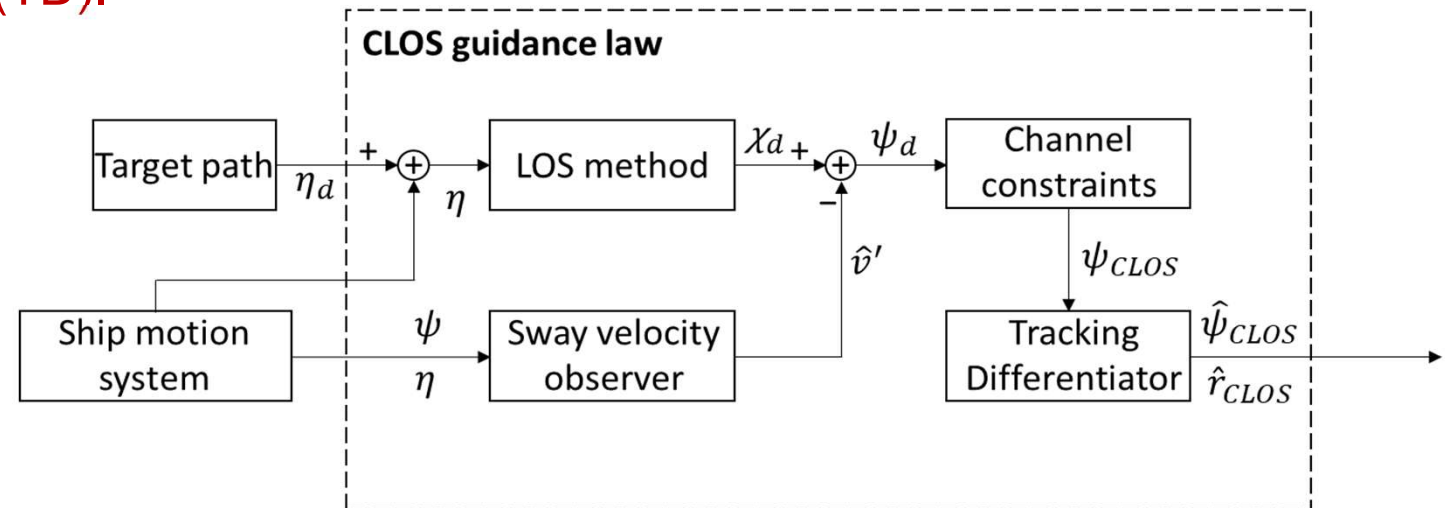
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3. CLOS guidance law



➤ Principle of the CLOS guidance law

- The proposed **constrained line-of-sight (CLOS) guidance law** includes four components: **LOS method**, **sway velocity observer**, **spatial constraints**, and **tracking differentiator (TD)**.



Flowchart of CLOS guidance law

3. CLOS guidance law



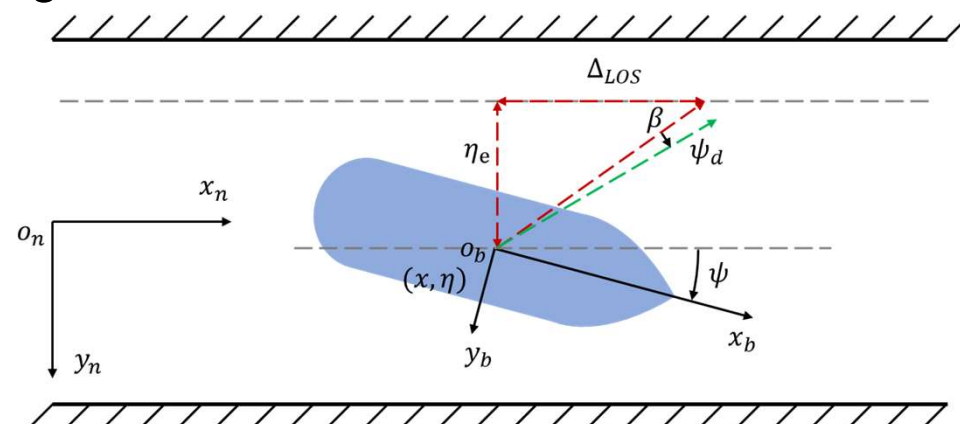
➤ LOS method

- The desired heading angle ψ_{LOS} is obtained according to the **LOS method** when considering the drift angle:

$$\psi_{LOS} = \arctan\left(\frac{\eta_e}{\Delta_{LOS}}\right) - \beta \quad \beta = \arctan(v/u) \approx v'$$

- The **look-ahead distance** Δ_{LOS} changes with the **water depth to draft ratios** and the **displacement** from the ship to center axis in the proposed method:

$$\Delta_{LOS} = \Delta_{max} + (\Delta_{max} - \Delta_{min})e^{-\lambda_1 \frac{h}{d} + \lambda_2 \eta'}$$



Schematic diagram of LOS guidance law

3. CLOS guidance law



➤ Sway velocity observer

- In order to **guarantee the accuracy of path following**, the sway velocity observer is utilized to estimate v' based on the known state η and ψ :

$$\begin{cases} \dot{\hat{\eta}} = \hat{v}' + \psi + \gamma_1 \tilde{\eta}' \\ \dot{\hat{v}}' = \gamma_2 \tilde{\eta}' \end{cases}$$

- Based on the values obtained from the sway velocity observer, the desired heading angle can be rewritten as follows:

$$\psi_{LOS} = \arctan\left(\frac{\eta_e}{\Delta_{LOS}}\right) - \beta \quad \longrightarrow \quad \psi_{LOS} = \arctan\left(\frac{\eta_e}{\Delta_{LOS}}\right) - \dot{\hat{v}}'$$

3. CLOS guidance law

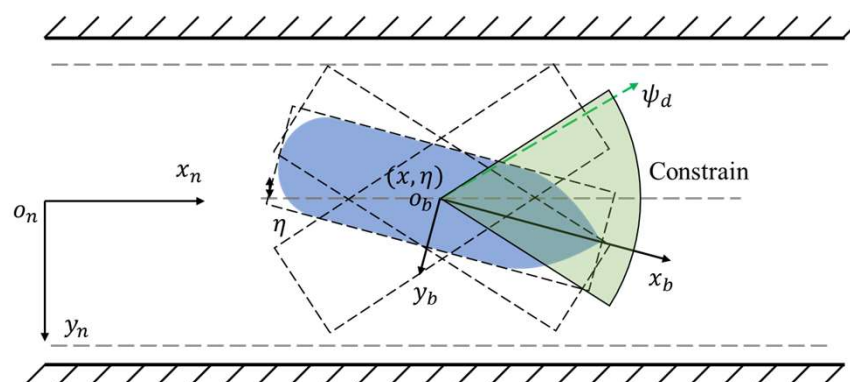


➤ Spatial constraints

- Based on the collision avoidance criteria, the **CLOS guidance law** is formulated to **guarantee safe navigation** as follows:

$$\psi_{CLOS} = \begin{cases} \psi_{LOS} & \text{if } f(\eta, \psi_{LOS}) < \frac{W}{2} - \kappa \\ \text{sgn}(\eta_e) \left[\arcsin\left(\frac{W - 2|\eta| - 2\tau}{\sqrt{L^2 + B^2}}\right) - \arctan\left(\frac{B}{L}\right) \right] & \text{else} \end{cases}$$

$$\tau = \kappa \left(1 - 2 \frac{|\eta|}{W - B} \right)$$



Schematic diagram of CLOS guidance law

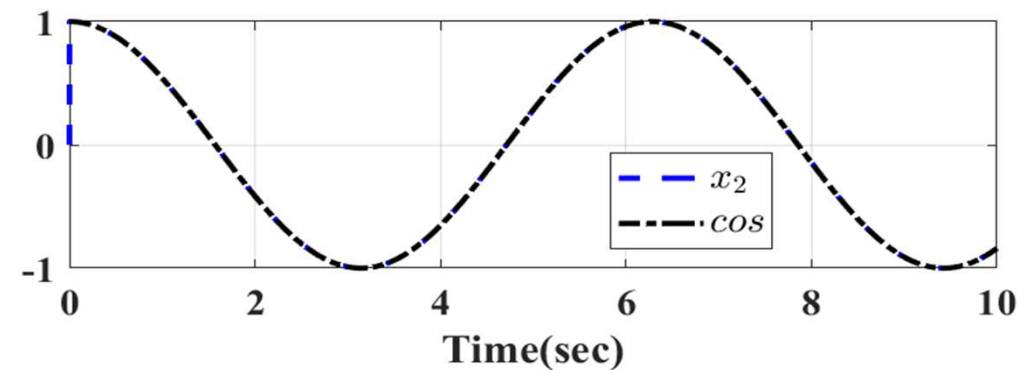
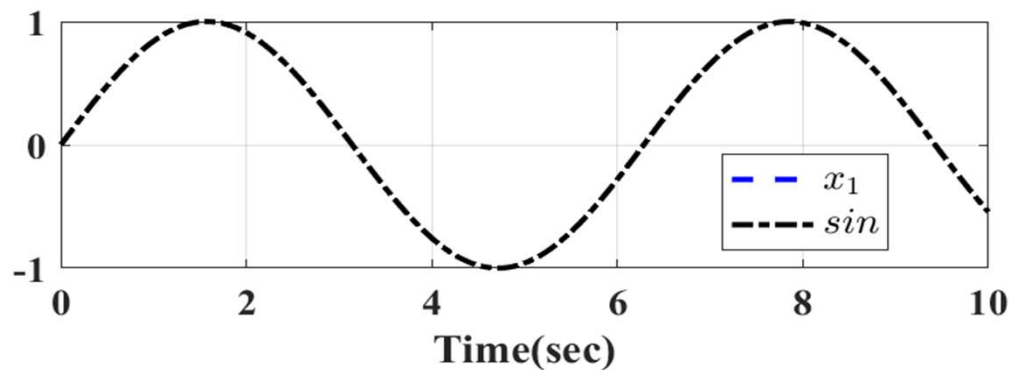
3. CLOS guidance law



➤ Tracking Differentiator (TD)

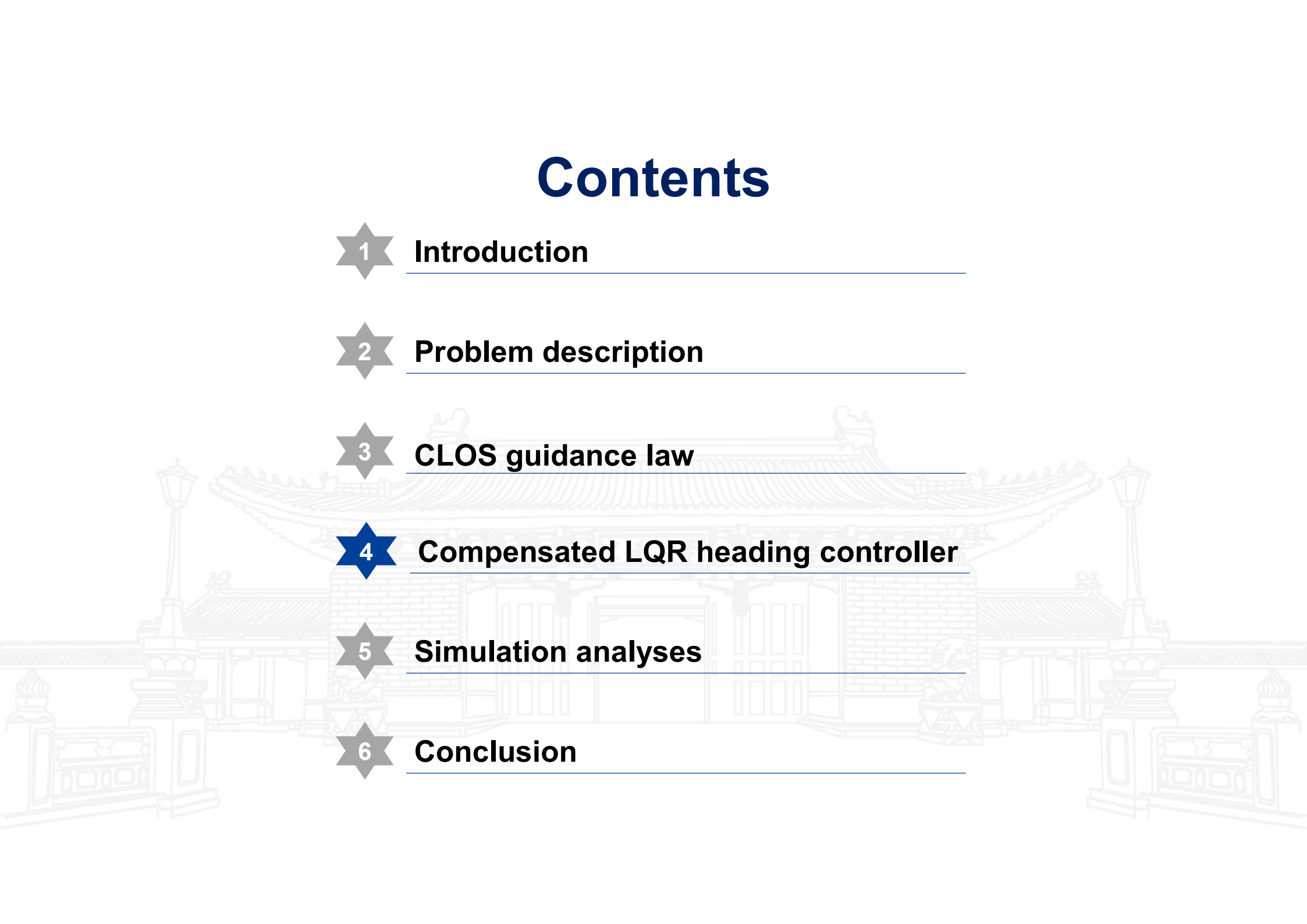
- A TD is employed to obtain the smoothed guidance heading angle and nondimensional guidance yaw rate:

$$\begin{cases} \hat{\psi}_{CLOS}(k+1) = \hat{\psi}_{CLOS}(k) + t_s \frac{U}{L} \hat{r}'_{CLOS}(k) \\ \hat{r}'_{CLOS}(k+1) = \hat{r}_{CLOS}(k) + t_s \frac{U}{L} \left[-q^2 (\hat{\psi}_{CLOS}(k) - \psi_{CLOS}(k+1)) - 2q \hat{r}_{CLOS}(k) \right] \end{cases}$$



Example of the tracking differentiator

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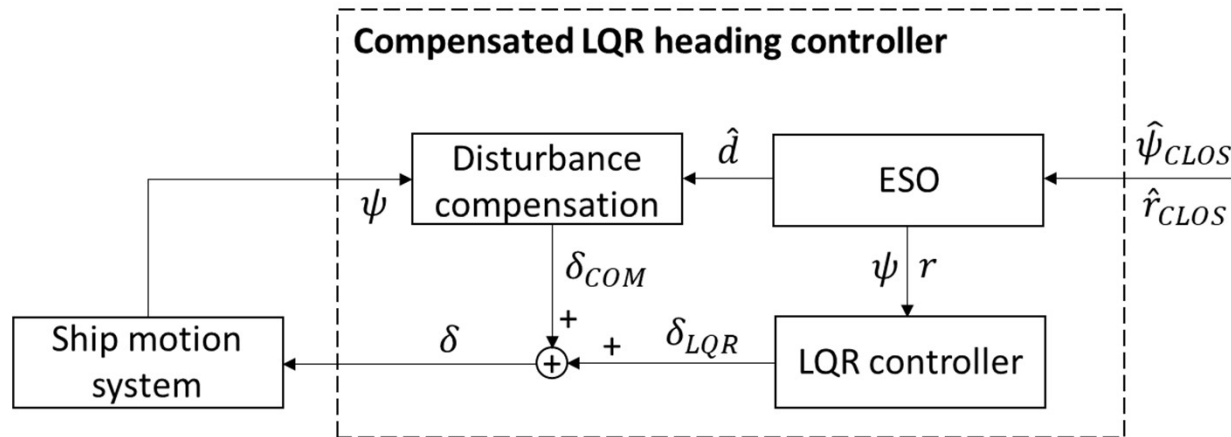
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4. Compensated LQR heading controller



➤ Principle of the compensated LQR heading controller

- The proposed compensated **linear quadratic regulator (LQR)** heading controller includes two components: **extended state observer (ESO)** and **compensated LQR control law**.



Flowchart of the Compensated LQR heading controller

4. Compensated LQR heading controller



➤ Nomoto model

- Considering that the yaw dynamics of ships can be described by the following **state space equation** based on the **Nomoto model**:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\delta + \mathbf{d} \quad \mathbf{x} = [\psi, r']^T$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{1}{T} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ \frac{K}{T} \end{bmatrix}, \quad \mathbf{d} = \begin{bmatrix} 0 \\ \xi \end{bmatrix}$$

- Based on the gain matrices, the control law can be formulated as follows:

$$\delta = -\mathbf{K}_1\mathbf{x} + \mathbf{K}_2\omega$$

4. Compensated LQR heading controller



➤ Extend State Observer (ESO)

- The **ESO** is utilized to estimate the yaw rate and the **total unknown term**:

$$\dot{\hat{\mathbf{x}}} = \bar{\mathbf{A}}\bar{\mathbf{x}} + \bar{\mathbf{B}}\delta + \boldsymbol{\mu}\tilde{\psi} \quad \bar{\mathbf{x}} = [\psi, r', \xi]^T$$

$$\bar{\mathbf{A}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{1}{T} & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad \bar{\mathbf{B}} = \begin{bmatrix} 0 \\ \frac{K}{T} \\ 0 \end{bmatrix}, \quad \boldsymbol{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix}$$

- Based on the estimation of ESO, the control law can be rewritten as follows:

$$\delta = \bar{\mathbf{K}}_1 \hat{\mathbf{x}} + \mathbf{K}_2 \omega$$

$$\omega = [\hat{\psi}_{CLOS}, \hat{r}'_{CLOS}]^T \quad \bar{\mathbf{K}}_1 = -[\mathbf{K}_1, \mathbf{K}_3]$$

4. Compensated LQR heading controller



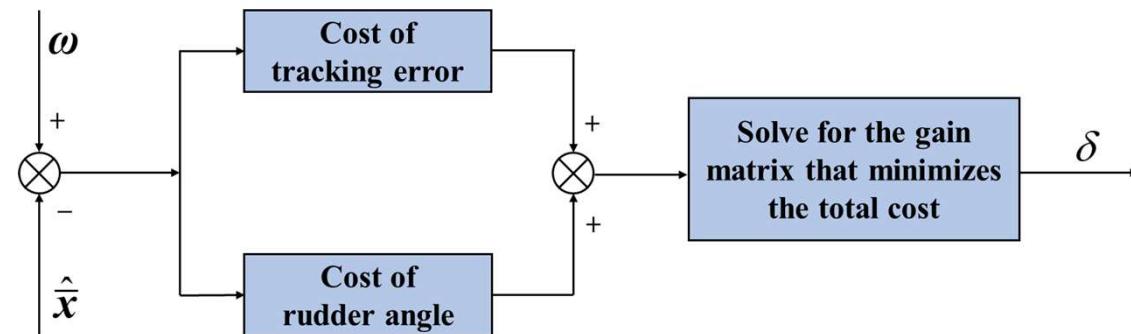
➤ LQR method

- Because the **yaw dynamics system** of the ship is controllable, the gain matrix K_1 is given as follows:

$$K_1 = \frac{1}{R} B^T P$$

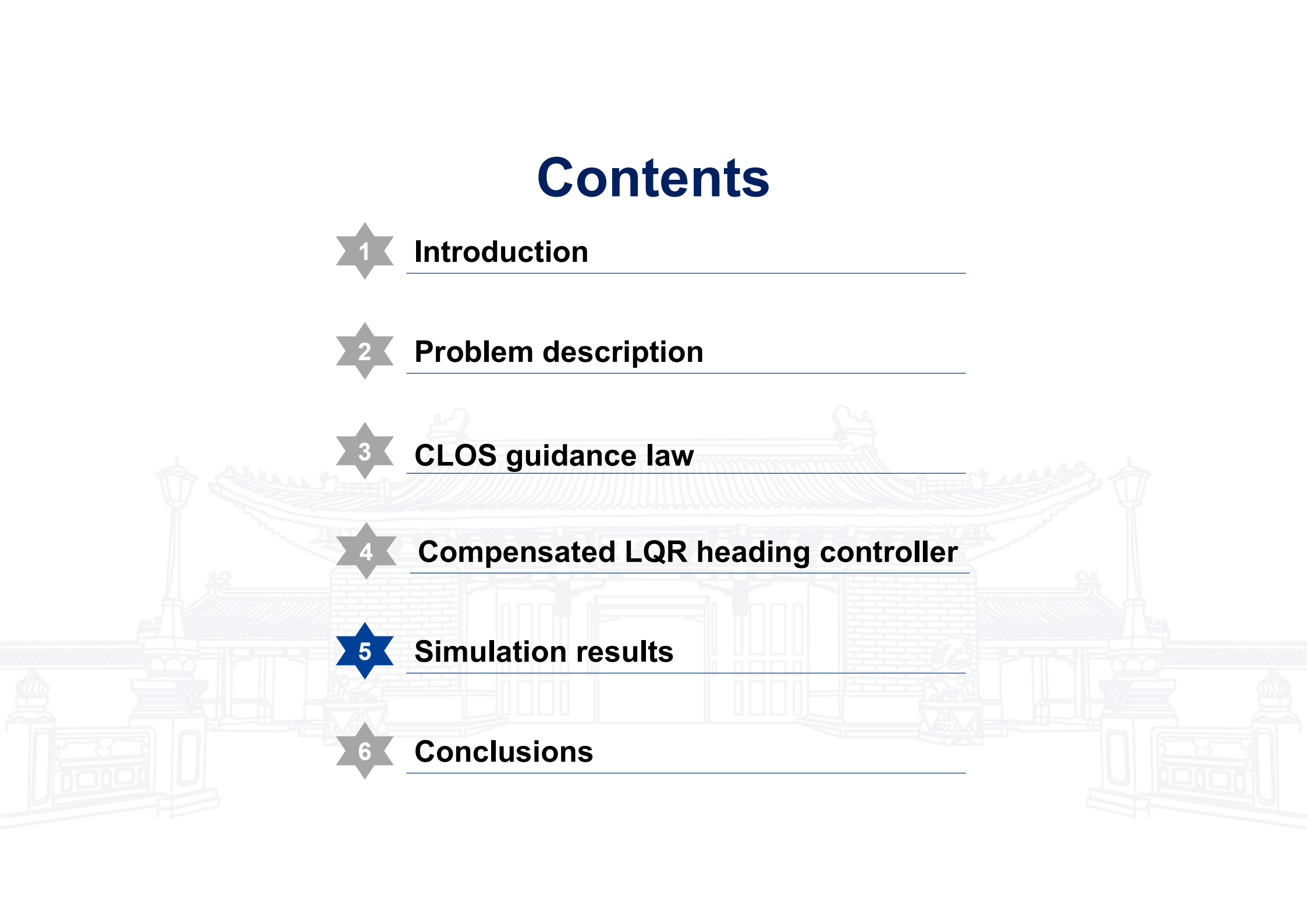
- K_2 and K_3 are selected as follows:

$$K_2 = -\left((A - BK_1)B\right)^\dagger \quad K_3 = \frac{T}{K}$$



Schematic diagram of LQR method

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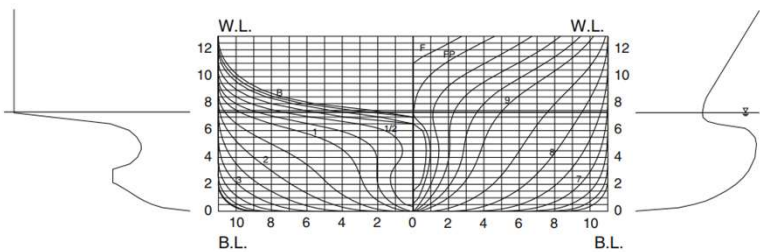
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5. Simulation results



➤ Selection of control parameters

Item	Symbol	Value
Length	L	136.7 m
Beam	B	22.0 m
Draft	d	7.36 m
Parameters of the sway velocity observer	γ_1	20
	γ_2	200
Parameters of the ESO	μ_1	24
	μ_2	192
	μ_3	512



Body plan and profiles
(Sano et al., 2014)

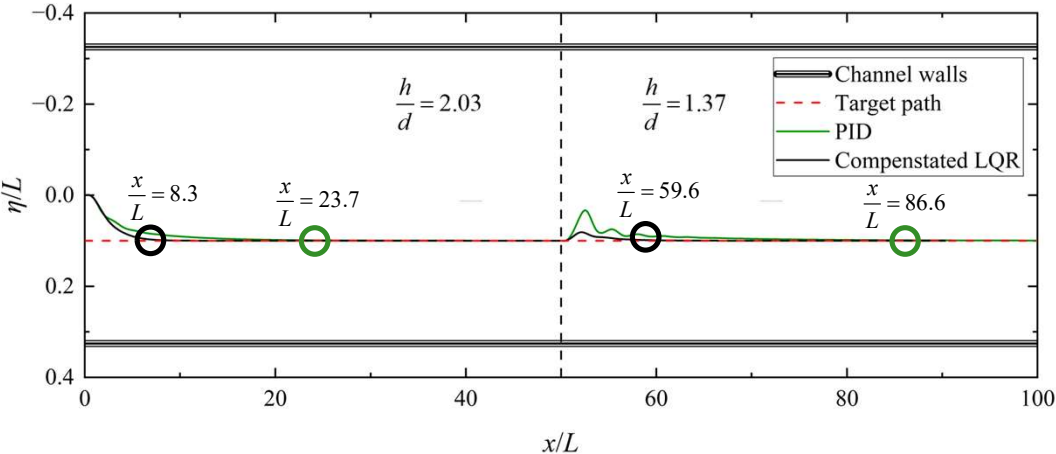


Side view of the ship model
(Sano et al., 2014)

5. Simulation results

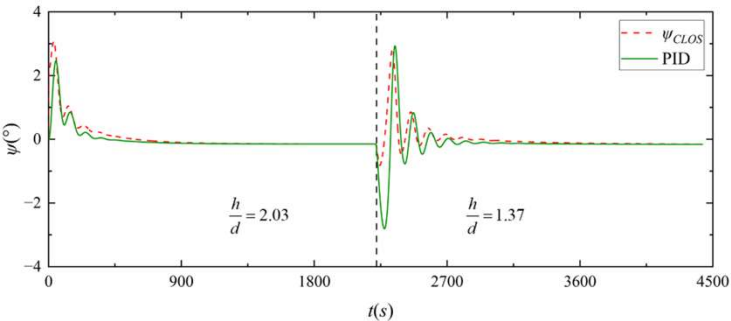


➤ Path following under restricted-water effects

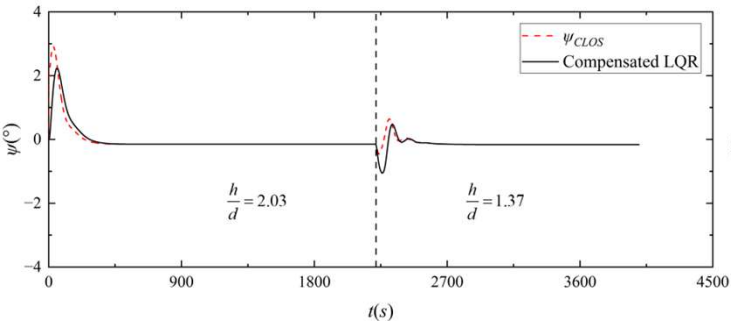


Target path and trajectory of the ship

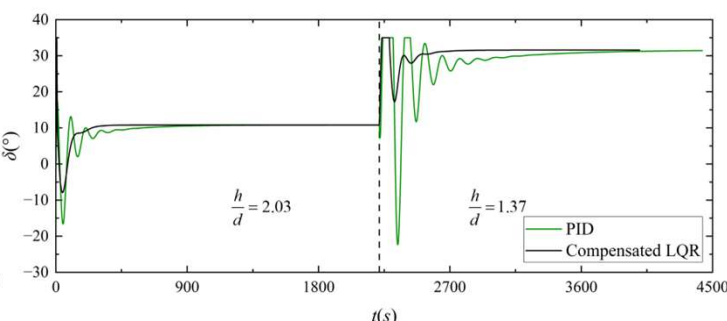
- The proposed method can make the ship track the target path more **rapidly**.
- The compensated LQR utilizes ESO to estimate disturbances, enabling **faster disturbance compensation** and **achieving superior control performance**.



Heading angle when using PID

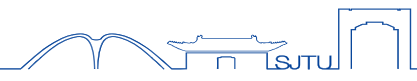


Heading angle when using compensated LQR

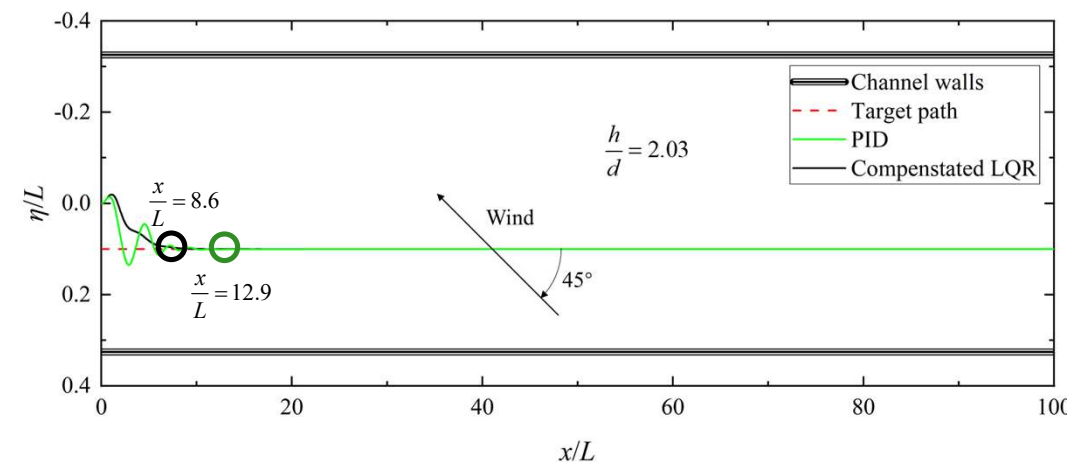


Rudder angle of the ship

5. Simulation results

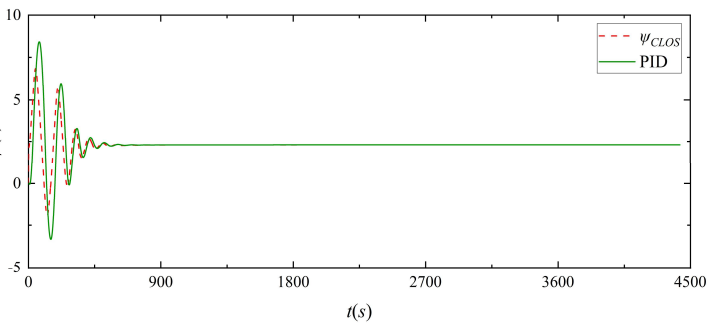


➤ Path following under restricted-water and wind effects

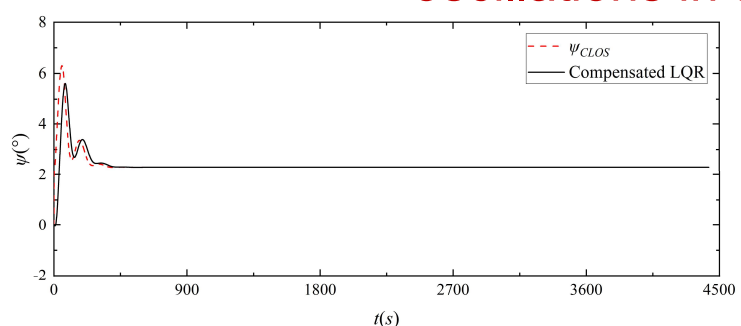


Target path and trajectory of the ship

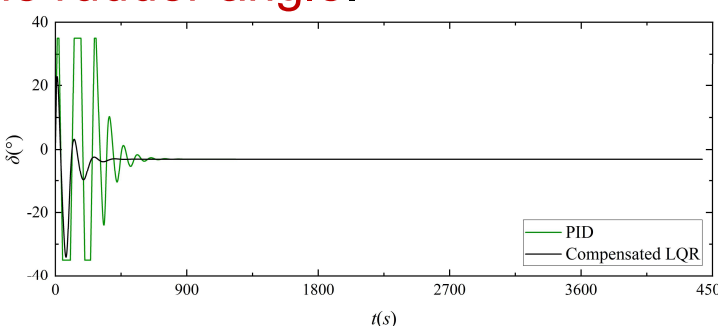
- The compensated LQR controller enables the ship to **smoothly and rapidly** converge to the desired path under restricted-water and wind effects.
- The proposed method results in **reduced oscillations in the rudder angle**.



Heading angle when using PID

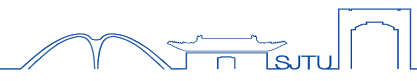


Heading angle when using compensated LQR

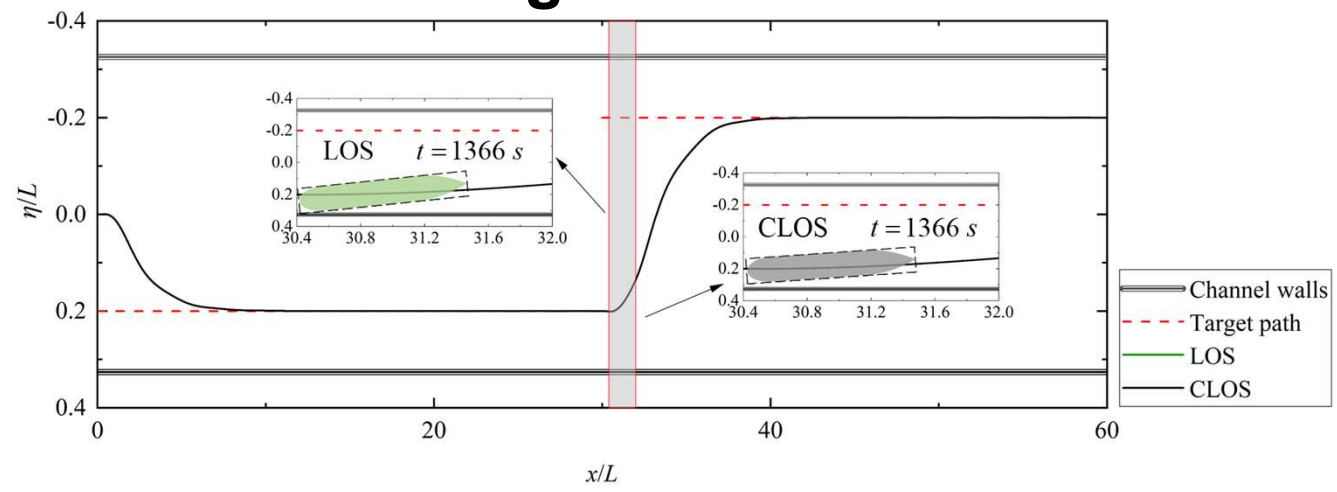


Rudder angle of the ship

5. Simulation results

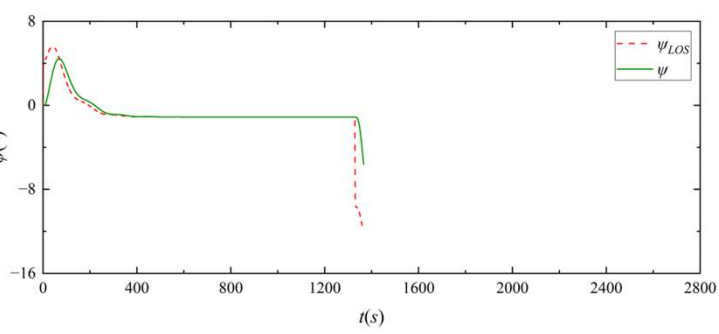


➤ Path following under collision-risk conditions

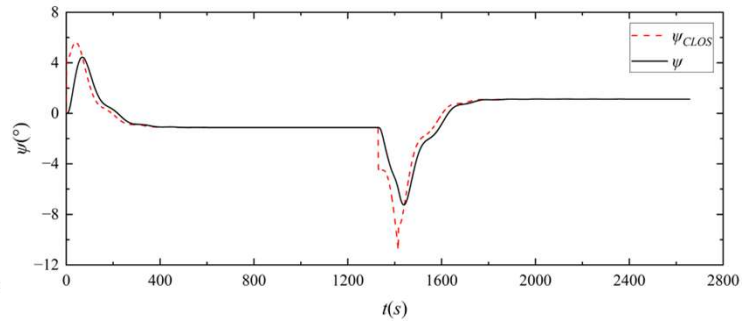


- The CLOS algorithm not only ensures precise tracking of the target path but also **guarantees collision avoidance with channel walls** throughout the tracking process.

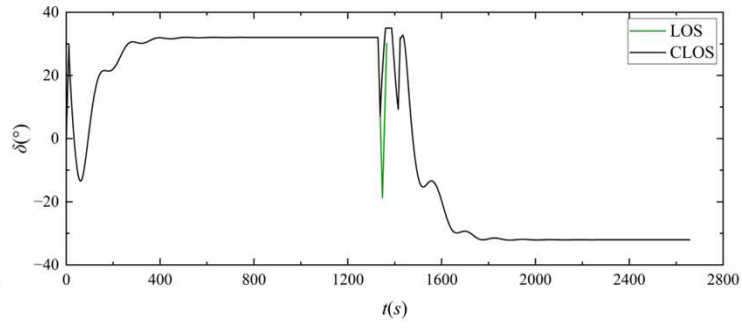
Target path and trajectory of the ship



Heading angle when using LOS

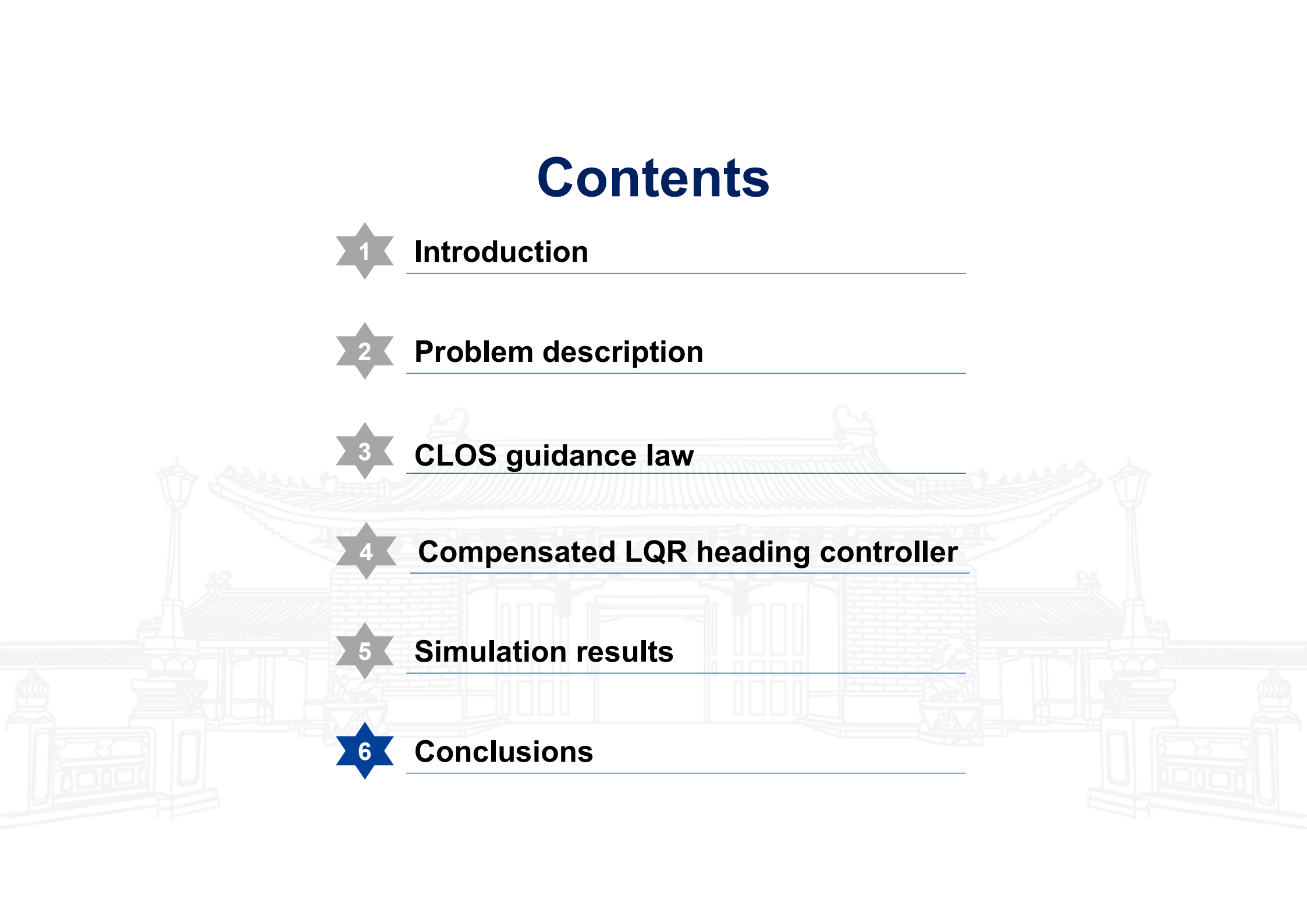


Heading angle when using compensated CLOS



Rudder angle of the ship

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5. Conclusions



- A controller integrating the **CLOS guidance law with the compensated LQR heading controller** is proposed for the path following in a narrow channel.
- The simulation results demonstrate that the proposed compensated LQR controller exhibits **significantly enhanced control performance** compared to the PID controller.
- It enables the ship to **track the target path more rapidly** in the channel, even under varying water depths as well as combined restricted water and wind effects.
- The proposed controller effectively ensures the ship to **avoid collisions** with sidewalls during the path-following task in the channel.



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